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The Diophantine Equations of Second and Higher Degree of the Form $3(xy-bx-ay+ab)=n(x+y-a-b)$ and $3(xy-bx-ay+ab)=n(x+y-a-b)$ Etc

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ABSTRACT:
In this paper some Diophantine equations of second and higher degree of the form $3(xy-bx-ay+ab)=n(x+y-a-b)$ and $3(xy-bx-ay+ab)=n(x+y-a-b)$ have been discussed where a, b and c are positive integers.

Key words: Diophantine equation, Egyptian fraction, conjecture and integral solution.

1 INTRODUCTION:

Erdos, P. & Straus, E. G. (1948) formulated the conjecture that for all integers $n \geq 2$, the rational number $\frac{4}{n}$ can be expressed as the sum of three unit fractions. The conjecture states that for every integer $n \geq 4$ there exists positive integers x, y and z such that

$$\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

These unit fractions form an Egyptian fraction representation of the number $\frac{4}{n}$. For example for n=1801, there exists solution x=451, y=295364 and z=3249004. The restriction on x, y and z to be positive make the problem difficult to solve. Computer searches have verified the truth of the conjecture upto $n \leq 10^{14}$ but proving it for all values of n remains an open problem.

The above relation can also be written as $4xyz = n(xy+yz+zx)$. As a polynomial equation with integer variables, this is a Diophantine equation. For values of n satisfying certain congruence relations, one can find an expansion for 4/n automatically as an instance of a polynomial identity. For instance, whenever n ≡ 2 (mod 3), 4/n has the expansion

$$\frac{4}{n}=\frac{1}{n}+\frac{1}{(n-2)/3+1}+\frac{1}{n((n-2)/3+1)}.$$

Here each of the three denominators n, (n − 2)/3 + 1, and n((n − 2)/3 + 1) is a polynomial of n, and each is an integer whenever n is 2 (mod 3).

Jaroma (2004) presented a three term solution with one negative term given by

$$\frac{4}{n}=\frac{1}{(n-1)/2}+\frac{1}{(n+1)/2}-\frac{1}{n(n-1)(n+1)/4}.$$

Hari Kishan, Megha Rani and Smiti Agarwal (2011) discussed the Diophantine equations of the form $3xy = n(x+y)$ and $3xyz = n(xy + yz + zx)$ etc.

In this paper, the Diophantine equations of second and higher degree of the form $3(xy - bx - ay + ab) = n(x + y - a - b)$ and $3(xy - bx - ay + ab) = n(x + y - a - b)$ etc. have been discussed where a, b and n are positive integers. Attempt has been made to obtain some integral solutions of these Diophantine equations.

2 Analysis:

(i) Quadratic Diophantine equation of the form $2(xy - bx - ay + ab) = n(x + y - a - b)$:

The given Diophantine equation can be written as

$$\frac{2}{n} = \frac{1}{x-a} + \frac{1}{y-b}. \quad \ldots (1)$$

The left hand side of the above equation can be written as

$$\frac{2}{n} = \frac{1}{\frac{(n-1)}{2}+1} + \frac{1}{n\left(\frac{(n-1)}{2}+1\right)}. \quad \ldots (2)$$

Comparing equations (1) and (2), we get $x = \frac{(n-1)}{2} + 1 + a$ and $y = n\left(\frac{(n-1)}{2}+1\right) + b$. Now if $n \equiv 1 (\mathrm{mod}\, 2)$ then x and y are positive integers. Thus these are the solutions of the above Diophantine equation.

Some solutions of the above Diophantine equation are as follows:

n	x	y
1	1+a	1+b
3	2+a	6+b
5	3+a	15+b
7	4+a	28+b
9	5+a	45+b
11	6+a	66+b
13	7+a	91+b
15	8+a	120+b
17	9+a	153+b

(ii) Quadratic Diophantine equation of the form $3(xy - bx - ay + ab) = n(x + y - a - b)$:

The given Diophantine equation can be written as

$$\frac{3}{n} = \frac{1}{x-a} + \frac{1}{y-b}. \quad \ldots (3)$$

The left hand side of the above equation can be written as

$$\frac{3}{n} = \frac{1}{\frac{(n-2)}{3}+1} + \frac{1}{n\left(\frac{(n-2)}{3}+1\right)}. \quad \ldots (4)$$

Comparing equations (3) and (4), we get $x=\frac{(n-2)}{3}+1+a$ and $y=n\left(\frac{(n-2)}{3}+1\right)+b$. Now if $n\equiv 2(\bmod 3)$ then x and y are positive integers. Thus these are the solutions of the above Diophantine equation.

Some solutions of the above Diophantine equation are as follows:

n	x	y
5	2+a	10+b
8	3+a	24+b
11	4+a	44+b
14	5+a	70+b
17	6+a	102+b
20	7+a	140+b
23	8+a	184+b
26	9+a	234+b
29	10+a	290+b

(iii) Cubic Diophantine equation of the form

$$3(xyz-ayz-bxz-cxy+abz+bcx+acy-abc)$$
$$=n(xy+yz+xz-bx-ay-cy-bz-az-cx+ab+bc+ac):$$

The given Diophantine equation can be written as

$$\frac{3}{n}=\frac{1}{x-a}+\frac{1}{y-b}+\frac{1}{z-c}. \qquad ...(5)$$

The left hand side of the above equation can be written as

$$\frac{3}{n}=\frac{1}{\frac{(n-2)}{3}+1}+\frac{1}{n\left(\frac{(n-2)}{3}+1\right)}$$
$$=\frac{1}{n}+\frac{1}{\frac{(n-1)}{2}+1}+\frac{1}{n\left(\frac{(n-1)}{2}+1\right)}. \qquad ...(6)$$

Comparing equations (5) and (6), we get $x=n+a$, $y=\frac{(n-1)}{2}+1+b$ and $z=n\left(\frac{(n-1)}{2}+1\right)+c$. Now if $n\equiv 1(\bmod 2)$ then x, y and z are positive integers. Thus these are the solutions of the above Diophantine equation.

Some solutions of the above Diophantine equation are as follows:

n	x	y	z
3	3+a	2+b	6+c
5	5+a	3+b	15+c
7	7+a	4+b	28+c
9	9+a	5+b	45+c
11	11+a	6+b	66+c
13	13+a	7+b	91+c
15	15+a	8+b	120+c

(iv) Cubic Diophantine equation of the form

$$4(xyz-ayz-bxz-cxy+abz+bcx+acy-abc)$$
$$=n(xy+yz+xz-bx-ay-cy-bz-az-cx+ab+bc+ac):$$

The given Diophantine equation can be written as

$$\frac{4}{n}=\frac{1}{x-a}+\frac{1}{y-b}+\frac{1}{z-c}. \qquad ...(7)$$

The left hand side of the above equation can be written as

$$\frac{4}{n}=\frac{1}{n}+\frac{1}{\frac{(n-2)}{3}+1}+\frac{1}{n\left(\frac{(n-2)}{3}+1\right)}. \qquad ...(8)$$

Comparing equations (7) and (8), we get $x=n+a$, $y=\frac{(n-2)}{3}+1+b$ and $z=n\left(\frac{(n-2)}{3}+1\right)+c$. Now if $n\equiv 2(\mathrm{mod}\,3)$ then x, y and z are positive integers. Thus these are the solutions of the above Diophantine equation.

Some solutions of the above Diophantine equation are as follows:

n	x	y	z
2	2+a	1+b	2+c
5	5+a	2+b	10+c
8	8+a	3+b	24+c
11	11+a	4+b	44+c
14	14+a	5+b	70+c
17	17+a	6+b	102+c
20	20+a	7+b	140+c

(v) Cubic Diophantine equation of the form

$$5(xyz-ayz-bxz-cxy+abz+bcx+acy-abc)$$
$$=n(xy+yz+xz-bx-ay-cy-bz-az-cx+ab+bc+ac):$$

The given Diophantine equation can be written as

$$\frac{5}{n}=\frac{1}{x-a}+\frac{1}{y-b}+\frac{1}{z-c}. \qquad ...(9)$$

The left hand side of the above equation can be written as

$$\frac{4}{5}=\frac{1}{n}+\frac{1}{\frac{(n-3)}{4}+1}+\frac{1}{n\left(\frac{(n-3)}{4}+1\right)}. \qquad ...(10)$$

Comparing equations (9) and (10), we get $x=n+a$, $y=\frac{(n-3)}{4}+1+b$ and $z=n\left(\frac{(n-3)}{4}+1\right)+c$. Now if $n\equiv 3(\mathrm{mod}\,4)$ then x, y and z are positive integers. Thus these are the solutions of the above Diophantine equation.

Some solutions of the above Diophantine equation are as follows:

n	x	y	z
3	3+a	1+b	3+c
7	7+a	2+b	14+c
11	11+a	3+b	33+c
15	15+a	4+b	60+c
19	19+a	5+b	95+c
23	23+a	6+b	138+c
27	27+a	7+b	189+c

(vi) Cubic Diophantine equation of the form

$$5(xyz - ayz - bxz - cxy + abz + bcx + acy - abc)$$
$$= n(xy + yz + xz - bx - ay - cy - bz - az - cx + ab + bc + ac):$$

The given Diophantine equation can be written as

$$\frac{6}{n} = \frac{1}{x-a} + \frac{1}{y-b} + \frac{1}{z-c}. \qquad ...(11)$$

The left hand side of the above equation can be written as

$$\frac{4}{6} = \frac{1}{n} + \frac{1}{\frac{(n-4)}{5}+1} + \frac{1}{n\left(\frac{(n-4)}{5}+1\right)}. \qquad ...(12)$$

Comparing equations (11) and (12), we get $x = n + a$, $y = \frac{(n-4)}{5} + 1 + b$ and $z = n\left(\frac{(n-4)}{5}+1\right) + c$. Now if $n \equiv 4 (\bmod 5)$ then x, y and z are positive integers. Thus these are the solutions of the above Diophantine equation.

Some solutions of the above Diophantine equation are as follows:

n	x	y	z
4	4+a	1+b	4+c
9	9+a	2+b	18+c
14	14+a	3+b	42+c
19-	19+a	4+b	76+c
24	24+a	5+b	120+c
29	29+a	6+b	174+c
34	34+a	7+b	238+c

3 Concluding Remarks: In this paper the solutions of some quadratic and cubic Diophantine equations of the form $2(xy - bx - ay + ab) = n(x + y - a - b)$ and $3(xyz - ayz - bxz - cxy + abz + bcx + acy - abc)$ $= n(xy + yz + xz - bx - ay - cy - bz - az - cx + ab + bc + ac)$ etc. have been obtained

These solutions have been shown in the respective tables.

Note: For a=0, b=0 and c =0, the results reduces to the results obtained by **Hari Kishan, Megha Rani and Smiti Agarwal** (2011).

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